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Virtual Manipulatives: What They Are and How Teachers Can Use Them

EMILY C. BOUCK AND SARA M. FLANAGAN



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Researchers and teachers have documented that many students with high-incidence disabilities struggle with mathematics, from automaticity of basic facts and computation to problem solving (Cawley, Parmar, Fley, Salmon, & Roy, 2001; Jitendra, DiPipi, & Perron-Jones, 2002; Montague, 1992; Woodward & Montague, 2002). Given the struggles that these students face in mathematics, researchers have attempted to review best, or evidence-based, practices for educating these students in this content domain (Maccini & Gagnon, 2000; Miller, Butler, & Lee, 1998). Manipulatives have been identified as a tool that repeatedly helps students with high-incidence disabilities succeed in mathematics (Maccini & Gagnon, 2000; Miller et al., 1998).

Mathematics manipulatives refer to concrete materials, that is, physical objects students can manipulate to explore and develop an understanding of a mathematical concept. Common manipulatives include chips, geoboards, cubes, square tiles, and tangrams. However, in the past decade computer-based virtual manipulatives

have been created that leverage innovations in technology, the Internet, software applications, and student appreciation of computers. Virtual manipulatives have been defined as an “interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” and are often modeled after concrete manipulatives (Moyer, Bolyard, & Spikell, 2002, p. 373). Important issues surrounding online virtual manipulatives include where they can be accessed (i.e., Web sites) and the possibility they hold in terms of educating students with high-incidence disabilities in mathematics.

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TABLE 1
Virtual Manipulative Available Online

| Virtual Manipulative Web sites | Web Addresses |
|---|---|
| National Library of Virtual Manipulatives | http://nlvm.usu.edu/ |
| eNLVM | http://enlvm.usu.edu/ |
| Illuminations | http://illuminations.nctm.org/ |
| <i>Interactivate</i> | http://www.shodor.org/interactivate/activities/ |
| MathTools | http://www.mathforum.org/mathtools/ |
| Arcytech | http://arcytech.org/java/ |

Virtual Manipulatives

Virtual manipulatives are defined as computer-based simulations of physical manipulatives that are accessed via the Internet or computer software. Examples include a virtual geoboard with virtual rubber bands, virtual Base 10 Blocks, and virtual square tiles. Virtual manipulatives are often similar to concrete manipulatives and often have the same names but are presented in an interactive manner through an online format or software environment. Students can perform almost any function with a virtual manipulative as they could with a concrete manipulative, such as using tiles to create rectangles and counting them to determine area and perimeter or stacking cubes to determine volume or surface area. However, virtual manipulatives have added interactive features (e.g., prompts that help students go onto the next step or feedback to answers) and provide students with unlimited quantities to use with a problem (Moyer et al., 2002). Although virtual manipulatives are relatively new, several Web sites have been developed to provide free virtual manipulatives for teachers to access and use with students.

Perhaps the most well-known virtual manipulative Web site is the National Library of Virtual Manipulatives (NLVM Cannon, Dorward, Duffin, & Heal [2004]; see Table 1). The NLVM was developed at Utah State University through National Science Foundation funding and has categorized its virtual manipulatives to align with the content standards of the National Council of Teachers of Mathematics (NCTM, 2000; i.e., numbers and operations, algebra, geometry, measurement, and data and probability) and grade levels (i.e., pre-K–2, 3–5, 6–8, and 9–12). These alignments help teachers better understand which virtual manipulatives offer the best models for representing the mathematical content at the appropriate developmental level.

One of the features unique to the NLVM is the teacher-control space (eNLVM). Using this site (registration is free), teachers can create their own virtual classroom. They can modify certain virtual manipulative activities, customizing them for the needs and ability level of their students or for particular problems they want their students to explore. This provides teachers with even greater control over the virtual manipulative, which allows them to harness virtual manipulatives for particular lessons as well as to clarify mathematical concepts that might be challenging for students. In addition, teachers can use the virtual manipulatives to help students solve their course and textbook problems. However, not all of the virtual manipulatives from the NLVM are modifiable, and some have varying degrees of modifiability.

In addition to the creating NLVM, the NCTM has developed a Web site with virtual manipulatives. The *Illuminations* Web site presents virtual manipulatives that are categorized by grade levels and allows teachers to search for lessons within these grade levels that use NCTM's online activities (i.e., lessons and virtual manipulatives). Another site, *Interactivate*, was developed by the Shodor Education Foundation (n.d.). *Interactivate* has more than 100 virtual manipulatives organized into the categories of numbers and operations, geometry, algebra, probability, statistics, modeling, discrete mathematics, and other.

Two other virtual manipulative Web sites are *MathTools*, from Drexel University (n.d.), and *Arcytech* (Bulaevsky, n.d.). The *MathTools* Web site is a compilation of mathematical tools and resources, including virtual manipulatives. To access the material on this site one needs to register, although access and use are free. *MathTools* also operates as a library, making available the manipulatives from the other sites (i.e., NLVM, Arcytech, *Interactivate*). *Arcytech* is a Web site created to allow teachers access to *applets* (small application programs) that can be used as an integral part of a mathematics curriculum. *Arcytech* contains the fewest virtual manipulatives (10) and focuses on money, pattern blocks, and materials for visualizing the ideas of the Pythagorean theorem.

The Potential of Virtual Manipulatives

We could not find any published research that specifically examined virtual manipulatives and students with high-incidence disabilities. However, the research from general education classes using online virtual manipulatives, of which some have included students with disabilities, is positive and suggests that virtual manipulatives are effective in facilitating students' understanding of

mathematical concepts (Reimer & Moyer, 2005; Steen, Brooks, & Lyon, 2006; Suh & Moyer, 2007; Suh, Moyer, & Heo, 2005). The positive results appear to be due to the visual nature of these manipulatives and students' ability to use them interactively (Reimer & Moyer, 2005; Suh et al., 2005). All of this suggests the potential of virtual manipulatives for educating and assisting students with disabilities in mathematics, but how?

How to Use Virtual Manipulatives

Virtual manipulatives can be used by teachers in similar ways as concrete manipulatives, such as (a) introducing or reviewing a mathematical idea, (b) developing understanding of mathematical concepts by visually representing those that are abstract, (c) scaffolding student learning, (d) and actively engaging students in learning (Moyer, 2001). Teachers can construct lessons around virtual manipulatives for their students or use the technology to reinforce concepts taught through other media. The virtual manipulatives outlined here can be used to teach, support, or review specific mathematical concepts: fractions, area and perimeter, place value, and combinations.

Fractions. Several virtual manipulatives focus on fraction concepts, such as representing, comparing, and adding or subtracting fractions. The NLVM has a particularly well-designed virtual manipulative for comparing fractions, *Fractions—Comparing* (http://nlvm.usu.edu/en/nav/frames_asid_159_g_3_t_1.html). It is just 1 of the 11 specific fraction-related virtual manipulatives in the NLVM and focuses on helping students determine which fraction is larger or smaller by renaming fractions so they have common denominators. With this applet, two different fractions appear on the screen, such as $\frac{2}{6}$ and $\frac{1}{2}$ and each fraction is represented with a part-to-whole circle or square model (see Figure 1). To determine the relationship between the two given fractions, students are prompted to rename each fraction so they have common denominators. Students do this by selecting an up or down arrow to change each fraction's denominator, which in turn makes the pieces in the model larger or smaller. As the denominator changes, the identified fractional part of the model (identified as a colored portion of the circle or square) remains the same but the number and size of the pieces change. Students can then see equivalent fractions, such as $\frac{4}{12}$ for $\frac{2}{6}$ and $\frac{6}{12}$ for $\frac{1}{2}$. From here, students still need to type the equivalent fractions into a textbox to answer the problem.

Students can check their answer by selecting a check box on the screen. If they are incorrect, students are

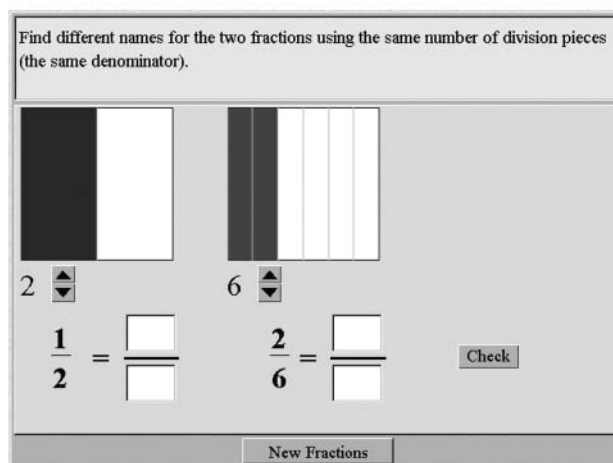


Figure 1. Comparing fraction manipulatives from the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/frames_asid_159_g_3_t_1.html). Reprinted by permission from the National Library of Virtual Manipulatives.

prompted to try again, with a suggestion to change the denominator or the numerator. If students are correct, a number line appears below the fractions—and students are prompted to locate and mark each of the fractions on the number line. After each fraction—either the original or the new equivalent ones—is placed on the number line, which is broken down into 12ths by the computer for this particular problem, students are urged to find a fraction that is between the two given fractions. In this case, students could select $\frac{5}{12}$. The applet indicates whether students are correct, and, if so, they are asked to find another fraction between the original two, which can be done by changing the number line divisions (i.e., 24ths).

There are many benefits of using virtual manipulatives to learn or reinforce fraction concepts. These benefits include (a) having a never-ending supply of virtual manipulatives available to students, (b) being able to quickly and easily work with more than one model (part-whole and location on a number line), and (c) not having to find or construct concrete fraction manipulatives for fractions that are difficult to model (i.e., 7ths or 11ths). In addition, virtual manipulatives allow students to receive immediate feedback if their answers are correct, along with a visual image that shows why.

Area and perimeter. Area and perimeter are two other mathematical concepts that are often better understood through the use of manipulatives, such as square tiles or “polyominoes.” Through the NLVM’s polyominoes virtual manipulative (http://nlvm.usu.edu/en/nav/frames_asid_115_g_3_t_3.html), students can investigate aspects of area and perimeter. Polyominoes, the online version

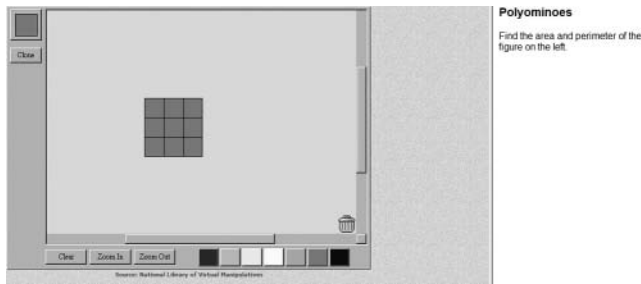


Figure 2. Polyominoes from the National Library of Virtual Manipulatives (http://www.nlvm.usu.edu/en/nav/frames_asid_115_g_3_t_3.html?open=activities&from=category_g_3_t_3.html). Reprinted by permission of the National Library of Virtual Manipulatives.

of concrete square tiles, allow students to construct figures for a given area or a given perimeter and then use the representation to visualize, count, and determine the other measure (e.g., given an area of 9, find a rectangle with the largest perimeter and one with the smallest perimeter; see Figure 2).

With the virtual manipulative polyominoes, teachers can develop student understanding of area and perimeter by having students cover or surround shapes to determine the amount inside a shape (area) or the length around a shape (perimeter). Students can be given bottom-edge and side-edge measures of rectangular figures and use the polyominoes to determine area and perimeter.

At the eNLVM Web site, the polyominoes virtual manipulative is very modifiable. It allows teachers to construct area and perimeter similar to what students would do with actual concrete tiles; yet this virtual manipulative eliminates common concrete manipulative concerns of not having enough materials, because students can participate with unlimited virtual tiles. The virtual polyominoes are easy for the student to move around, delete if she or he wants to start over, and even change colors to make each tile easier to see.

Place value. The virtual manipulatives discussed so far have been for upper elementary and middle school students, but not all virtual manipulatives are for older students. There are many virtual manipulatives for younger students or students who operate at a younger age in terms of mathematical development. Two such virtual manipulatives available for younger students are Base 10 Blocks, which can be used to develop understanding of place value as well as addition and subtraction operations, and virtual manipulatives that introduce the idea of combinations. The NLVM has several place-value activities in which students can represent numbers with Base 10 Blocks as well as engage in activities that address addition and subtraction of whole numbers. There

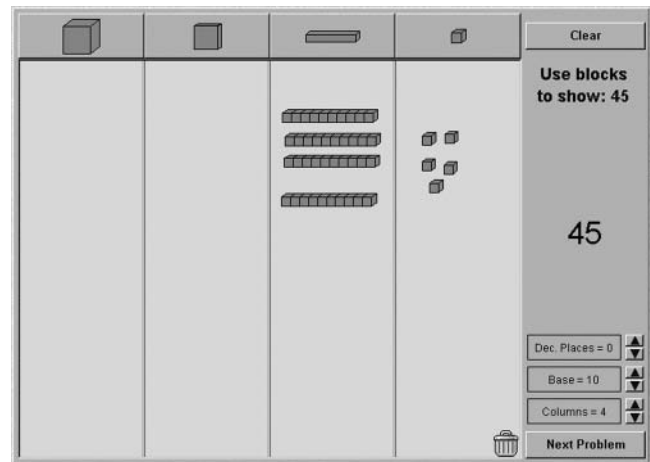


Figure 3. Base 10 Blocks manipulative from the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/frames_asid_152_g_2_t_1.html). Reprinted by permission of the National Library of Virtual Manipulatives.

are also activities with Base 10 Blocks for decimal place value and addition for older students.

With the NLVM's virtual Base 10 Blocks, students are prompted to build numbers. For example, students are asked to show the number 45 using the Base 10 Blocks: four 10-unit blocks (i.e., rectangular blocks that are 1 unit by 1 unit by 10 units) and five 1-unit blocks (i.e., 1 unit by 1 unit by 1 unit; see Figure 3). The blocks can also be used to show and solve addition problems. With virtual Base 10 Blocks; students can represent each addend, determine a sum, state a solution, and check their answers.

One benefit of the virtual Base 10 Blocks is that when a student groups blocks together to form clusters of 10s or 100s, the screen shows the blocks coming together to help students visualize the process. Similar to the other virtual manipulatives discussed, the Base 10 Blocks are highly interactive, which provides a level of fun and interest for students.

Combinations. Another example of a virtual manipulative for younger students is Bobbie Bear from the NCTM Illuminations Web site, which addresses the idea of combinations. In this virtual manipulative, students develop combinations of clothing for the Bobbie Bear character from a closet containing three different-colored shirts and three different-colored pants (see Figure 4). Students first guess the number of variations that Bobbie Bear can wear and then create the different outfit combinations. Combinations of clothing are displayed so students can keep track of which combinations they have made. When students have made all combinations possible,

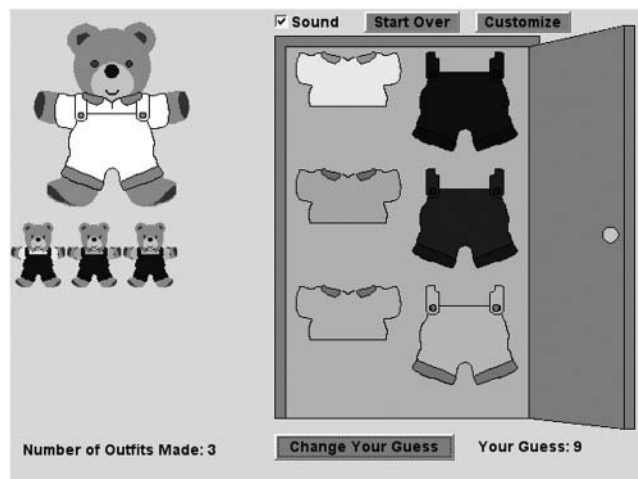


Figure 4. Bobbie Bear from the National Council of Teachers of Mathematics Illuminations site (<http://illuminations.nctm.org/ActivityDetail.aspx?ID=3>). Reprinted by permission from the National Council of Teachers of Mathematics.

the closet door shuts. The number of shirts and pants options for Bobbie Bear can be customized with up to five different-colored shirts and four different-colored pants. This virtual manipulative has an auditory feedback feature, which gives students immediate reinforcement when they have made a new, correct combination.

The Appeal of Virtual Manipulatives

An important appeal of virtual manipulatives for any teacher, and particularly a special education teacher, is the range of manipulatives that one can access, especially when resources in schools are limited. For example, many online virtual manipulatives are free and have been created to address a variety of mathematical concepts at various ages and developmental levels. Most concrete manipulatives a teacher might want to incorporate into a lesson, such as tiles, cubes, and geoboards, are available as virtual manipulatives; thus, little change is needed to lesson planning aside from inclusion of technology. With virtual manipulatives, a teacher can demonstrate a mathematical concept to the whole class or have students work individually or with partners at computers.

Added to this is the opportunity for teachers to customize some of the virtual manipulatives so that students are working on problems similar to those included in their textbooks. With this technology, a teacher can have a student who is working at a different mathematical level work independently using the virtual manipulatives to help him or her understand a particular mathematical concept while the teacher works on a different concept

with another group of students. This allows students to work on different mathematical lessons using different virtual manipulatives tailored to each student's mathematical ability. Given the number and range of virtual manipulatives, teachers can easily differentiate instruction for the students within their classroom.

For students, the appeal of the virtual manipulatives comes with the interactive, engaging, and immediate feedback features of the tool. Letting students know immediately whether their answer is correct is not only helpful but also allows students to be more independent of the teacher. Students can also work at their own pace with online virtual manipulatives and, if they have access to the Internet outside of school, can use these same tools to support their homework or studying.

The Challenge With Virtual Manipulatives

Virtual manipulatives, which are exciting and interactive, are based on technology. To incorporate online virtual manipulatives into their teaching, teachers need to familiarize themselves with the different Web sites, research the different activities within those sites, and determine which activities are appropriate for their curriculum and their students' skill levels. Both virtual and concrete manipulatives are only beneficial tools if teachers know how to integrate them into their teaching.

Conclusion

Research on the positive impact of using concrete manipulatives in mathematics for students with high-incidence disabilities is clear. Maccini and Gagnon (2000) considered manipulatives to be a best practice in terms of educating students with high-incidence disabilities in mathematics. It would follow, then, that research on virtual manipulatives may also produce positive results for students with high-incidence disabilities. Although virtual manipulatives present some challenges, special educators need to consider manipulatives as a means of helping their students learn mathematics and should be open to the use of virtual manipulatives. Teachers can begin by taking lessons on specific concepts, such as area and perimeter, and using virtual manipulatives in place of concrete manipulatives. Teachers can also seek out the knowledge and advice of highly qualified general education mathematics teachers who can work with teachers to connect mathematical ideas and assess the strengths

and limitations of a particular manipulative. As teachers become more confident with virtual manipulatives, they can construct new lessons that expand student experiences through use of the multiple opportunities that the sites afford as well as the multiple types of manipulatives.

Virtual manipulatives have the potential to benefit teachers and students in mathematics teaching and learning. Students' interest in computers and the accompanying motivation can be captured with virtual manipulatives, and teachers can take advantage of their students' increasing ability to use this technology. To better understand these new educational tools, teachers should try virtual manipulatives in their classrooms, researchers should study the use of virtual manipulatives in educating students with disabilities, and teacher educators should prepare preservice and in-service teachers to implement this technology with their students.

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References

- Bulaevsky, J. (n.d.). *Arcytech*. Retrieved October 4, 2007, from <http://arcytech.org/java/>
- Cannon, L., Dorward, J., Duffin, J., & Heal, B. (2004). National Library of Virtual Manipulatives. *The Mathematics Teacher*, 97, 158-159.
- Cawley, J., Parmar, R., Fley, T. E., Salmon, S., & Roy, S. (2001). Arithmetic performance of students: Implications for standards and programming. *Exceptional Children*, 67, 311-328.
- Drexel University. (n.d.). *MathTools*. Retrieved October 4, 2007, from <http://www.mathforum.org/mathtools/>
- eNLVM. (n.d.). *National Library of Virtual Manipulatives*. Retrieved November 12, 2007, from <http://enlvm.usu.edu/ma/nav/index.jsp>
- Jitendra, A., DiPipi, C. M., & Perron-Jones, N. (2002). An exploratory study of schema-based word-problem-solving instruction for middle school students with learning disabilities: An emphasis on conceptual and procedural understanding. *Journal of Special Education*, 36, 23-38.
- Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children*, 32(5), 1-22.
- Miller, S. P., Butler, F. M., & Lee, K. (1998). Validated practices for teaching mathematics to students with learning disabilities: A review of the literature. *Focus on Exceptional Children*, 31(1), 1-24.
- Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem-solving of middle school students with learning disabilities. *Journal of Learning Disabilities*, 25, 230-248.
- Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, 47, 175-197.
- Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics*, 8, 372-377.
- National Council of Teachers of Mathematics. (n.d.). *Illuminations*. Retrieved October 4, 2007, from <http://illuminations.nctm.org/ActivitySearch.aspx>
- National Council of Teachers of Mathematics. (2000). *Principles and NCTM standards for school mathematics*. Reston, VA: Author.
- National Library of Virtual Manipulatives. (n.d.). *National Library of Virtual Manipulatives*. Retrieved October 4, 2007, from <http://nlvm.usu.edu/en/nav/vlibrary.html>
- Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching*, 24, 5-25.
- Steen, K., Brooks, D., & Lyon, T. (2006). The impact of virtual manipulatives on first grade geometry instruction and learning. *Journal of Computers in Mathematics and Science Teaching*, 24, 373-391.
- Shodor Education Foundation. (n.d.). *Interactivate*. Retrieved October 4, 2007, from <http://www.shodor.org/interactivate/activities/>
- Suh, J., & Moyer, P. S. (2007). Developing students' representational fluency using virtual and physical algebra balances. *Journal of Computers in Mathematics and Science Teaching*, 26, 155-173.
- Suh, J., Moyer, P. S., & Heo, H.-J. (2005). Examining technology uses in the classroom: Developing fraction sense using virtual manipulatives concept tutorials. *Journal of Interactive Online Learning*, 3(4), 1-21.
- Woodward, J., & Montague, M. (2002). Meeting the challenge of mathematics reform for students with LD. *Journal of Special Education*, 36, 89-101.